Mac-cormack Flux Corrected Transport Scheme for Simulation of Dam Break and Super Critical Flow in Curvilinear Coordinate System

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Abstract. Dam break is one of a hydraulic phenomenon can cause material and life losses to the downstream. Dam break generally caused by geotechnical failures such as earthquakes, excessive pore water pressure, and materials used in dam construction. The use of mathematical models in the simulation of diverse hydraulic phenomena has become essential as a predictive tool in the evaluation of proposed engineering works to analyze the risk of disaster caused by dam failure or other hydraulic phenomena such as supercritical flow that has high impact and velocity, so the damage and losses caused by dam failure can be reduced. All hydraulic phenomena are naturally three-dimensional and unsteady, but computation involving assumptions of one-dimensionality or quasi-two dimensionality and/or steadiness have been successfully applied. A major disadvantage of Finite difference Method is that, when using a rectangular grid network, irregular physical boundaries must be treated using a "stair stepped" representation. A better representation of the physical boundary can be generation of a curvilinear coordinate system with coordinate lines coincident with the generally irregular physical boundaries. This paper present application of the Mac-Cormack scheme to solve the St Venant equations in generalized coordinates describing the depth-averaged, unsteady, sub and super-critical free-surface flows and hydraulic jumps. The calculated results are compared to experimental or analytical data. Comparisons with analytics as well as with other numerical solutions show that the proposed method is relatively accurate, fast, and reliable.

Keywords **:** Dam Break, Supercritical flow, Mac-Cormack FCT Scheme, Saint-Venant Equation, Finite Difference Method

1. Introduction

Dam break is one of a very dangerous hydraulic phenomena that can cause a lot of material and life losses to the downstream of the dam. Dam break event suddenly generates a large mass of water to the downstream due to the failure of the dam construction, causing a severe flood at the downstream of the dam. Dam break generally caused by geotechnical failures such as earthquakes, excessive pore water pressure, and materials used in dam construction (T. R. Maitsa 2019). Therefore, it is necessary to analyze the risk of the dam break event for disaster mitigation, so the impact of this event can be minimized.
The use of mathematical models in the simulation of diverse hydraulic phenomena has become essential as a predictive tool in the evaluation of proposed engineering works to analyze the risk of disaster caused by dam failure or other hydraulic phenomena such as supercritical flow that has high impact and velocity, so the damage and losses caused by dam failure can be reduced. Shallow water Equation (SWE) or usually known as Saint-Venant Equation (SVE) is the common method to describe Dam Break problem mathematically (C.T. Hsu 2002). The equation can be solved using numerical method by Finite Difference the least complicated method. This paper present application of the Mac-Cormack scheme to solve the St Venant equations in generalized coordinates describing the depth-averaged, unsteady, sub and super-critical free-surface flows and hydraulic jumps.

2. Formatting the title, authors and affiliations

Please In this paper the governing equation is St. Venant Equation (SVE) and numerical scheme is using Mac-Cormack Flux Corrected Transport (FCT) Scheme. The flow chart of the study is given by Figure.1 below.

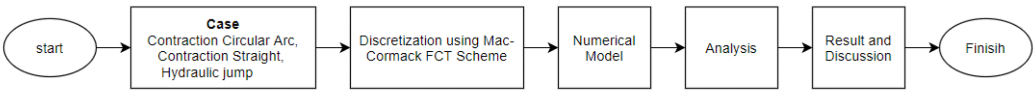

Figure 1. Flow chart

The SVE is based on the assumptions: uniform velocity distribution, incompressible fluid, hydrostatic pressure, and small channel bottom slope. The SVE consists of the continuity and momentum equations as given as bellow.

- Continuity Equation

$$\frac{\partial}{\partial t}[H] + \frac{\partial}{\partial x}[HU] + \frac{\partial}{\partial y}[HV] = 0 \tag{1}$$

- Momentum Equation
  - In the x- direction

$$\frac{\partial}{\partial t}[UH] + \frac{\partial}{\partial x}\left[U^2H + \frac{1}{2}gH^2\right] + \frac{\partial}{\partial y}[UVH] = [-gH(S_{0x} - S_{fx})] \tag{2.a}$$

  - In the y-direction

$$\frac{\partial}{\partial t}[VH] + \frac{\partial}{\partial x}[UVH] + \frac{\partial}{\partial y}\left[V^2H + \frac{1}{2}gH^2\right] = [-gH(S_{0y} - S_{fy})] \tag{2.b}$$

In Cartesian and conservative vector, the SVE eq. is given as follows.

$$\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = S \tag{3}$$

$$Q = \begin{bmatrix} H \\ UH \\ VH \end{bmatrix} \tag{4.a}$$

$$F = \begin{bmatrix} UH \\ U^2H + \frac{1}{2}gH^2 \\ UVH \end{bmatrix} \quad (4.b)$$

$$G = \begin{bmatrix} VH \\ UVH \\ V^2H + \frac{1}{2}gH^2 \end{bmatrix} \quad (4.c)$$

$$S = \begin{bmatrix} 0 \\ -gH(S_{0x} - S_{fx}) \\ -gH(S_{0y} - S_{fy}) \end{bmatrix} \quad (4.d)$$

$$S_{fx} = \frac{n^2U\sqrt{U^2+V^2}}{h^{4/3}}; \qquad S_{fy} = \frac{n^2V\sqrt{U^2+V^2}}{h^{4/3}} \quad (5)$$

Where:

H = water surface elevation
U = velocity in the x-direction
V = velocity in the y-direction
g = gravity
S_{0x} and S_{0y} = bottom channel slope in the x and y direction
S_{fx} and S_{fy} = bottom channel friction in the x and y direction

To be able to solve the discretization using **finite difference**, it is necessary to transform the governing equation into Curvilinear Coordinate (computational space ξ *and* η) from the Cartesian Coordinate (x and y). In this case, the curvilinear grid method is used with the following relationship bellow.

$$\varsigma \text{ (sigma)} = t$$
$$\xi \text{ (xi)} = \xi(t, x, y)$$
$$\eta \text{ (eta)} = \eta(t, x, y) \quad (6)$$

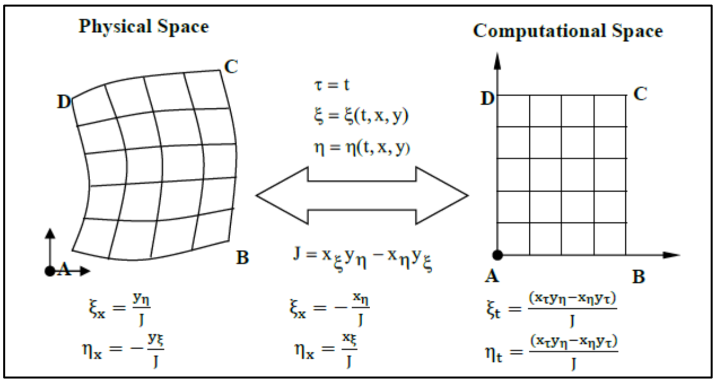

Figure. 1. Generalized Curvilinear Coordinate Transformations

Coordinate transformation is done to uniform the space in physical space into computational space. To represent the Cartesian derivatives ∂x and ∂y, in terms of the curvilinear derivatives, **Chain rule expansions are used**. By reversing the role of the independent variables in the chain rule formulas, curvilinear derivatives $\partial\xi$ and $\partial\eta$ in terms of the Cartesian derivatives. By solving Equation for the curvilinear derivatives in terms of the Cartesian derivatives yields the following metrics.

$$\xi_x = \frac{y_\eta}{J} \quad \xi_x = -\frac{x_\eta}{J} \quad \xi_t = \frac{(x_\tau y_\eta - x_\eta y_\tau)}{J}$$
$$\eta_x = -\frac{y_\xi}{J}; \quad \eta_x = \frac{x_\xi}{J}; \quad \eta_t = \frac{(x_\tau y_\eta - x_\eta y_\tau)}{J} \quad (7)$$

where

$$J = x_\xi y_\eta - x_\eta y_\xi \tag{8}$$

J is Jacobian of transformation. The value of ξ_t and η_t are zero for cases that doesn't involve grid movement. By providing the x, y coordinate (Cartesian) of grid points, the metrics ξ_t, η_t, ξ_x, η_x, ξ_y and η_y can be generate numerically using finite difference approximations. A full derivation of the transformed equations is given in (Ari 2001).
In curvilinear coordinates and conservative vector form, neglecting the depth-averaging stresses, so the SVE are as follows.

$$\frac{\partial \overline{\boldsymbol{Q}}}{\partial \tau} + \frac{\partial \overline{\boldsymbol{F}}}{\partial \xi} + \frac{\partial \overline{\boldsymbol{G}}}{\partial \eta} = S \tag{9}$$

Where:

$$\overline{Q} = \frac{Q}{J}\,; \qquad \overline{F} = \frac{1}{J}\left[\xi_x F + \xi_y G\right]\,; \quad \overline{G} = \frac{1}{J}\left[\eta_x F + \eta_y G\right] \tag{10}$$

$$S \;= -gh\begin{bmatrix} 0 \\ \frac{(y_\eta S_{0\xi} - y_\xi S_{0\eta})}{J} - S_{fx} \\ \frac{(x_\eta S_{0\xi} - x_\xi S_{0\eta})}{J} - S_{fy} \end{bmatrix} \tag{11}$$

Where:

$$U = u\frac{\partial \xi}{\partial x} + v\frac{\partial \xi}{\partial y}\,\textit{(x direction)};\quad V = u\frac{\partial \eta}{\partial x} + v\frac{\partial \eta}{\partial y}\,\textit{(y-direction)} \tag{12}$$

The explicit finite-difference method used in this paper is based on Mac-Cormack Scheme (1982). The explicit version of the scheme is popular for dealing with one-, and two-dimensional unsteady open channel flows.
Although the derivatives in equation (3) are discretized to the first order of accuracy, it is claimed that the scheme is of the second order of accuracy in both space and time. It is also claimed that it doesn't require shock-fitting procedure, particularly suitable for solving hydraulic jump problems in open channels. The **Mac-Cormack explicit finite-difference scheme consists of a twostep predictor-corrector sequence.**

- Predictor step: (using forward step)
$$Q_{i,j}^P = Q_{i,j}^k - \frac{\Delta t}{\Delta \xi}\nabla_\xi F_{i,j}^k - \frac{\Delta t}{\Delta \eta}\nabla_\eta G_{i,j}^k - \Delta t S_{i,j}^k \tag{13}$$
- Corrector step: (using backward step)
$$Q_{i,j}^C = Q_{i,j}^k - \frac{\Delta t}{\Delta \xi}\Delta_\xi F_{i,j}^P - \frac{\Delta t}{\Delta \eta}\Delta_\eta G_{i,j}^P - \Delta t S_{i,j}^P \tag{14}$$
- Solution step:
$$Q_{i,j}^{k+1} = Q_{i,j}^k - \frac{1}{2}(Q_{i,j}^P + Q_{i,j}^C) \tag{15}$$

3. Result and Discussion

3.1. Supercritical Flow in Circular-Arc Wall Contraction

Although the derivatives in equation (3) are discretized to the first order of accuracy, it is claimed that

The first case of supercritical flow is Circular-Arc contraction from "Open Channel Flow" written by M. Hanif Chaudhry in 1993. This case has data as follows:

- Froude number (Fr) = 4
- Initial water surface (H_0) = 0.030 m
- Manning (n) = 0.012

- Slope (S_0) = 0.07

From the data can be obtained the parameters needed for input on the program as follows.

- Gravitation (g) = 9.81 m/s^2
- Flow velocity in x- direction (u_0)
  $U_0 = F \cdot \sqrt{g \cdot H_0}$ (U_0) = 2.16998 m/s
- Flow velocity in y-direction (V_0) = 0.0 m/s

In the computations, a constant depth and uniform velocity distribution were `assumed at the upstream section and 71 grid point in *x*-direction and 41 grid point in *y*-direction. Computation time for this model are 0.01 second for time step (Δt), 100 for the interval, and 10 second for the total simulation time. The results are as follows .

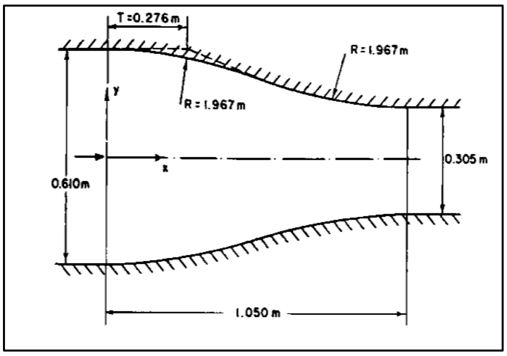

Figure 2. Profile Circular-arc

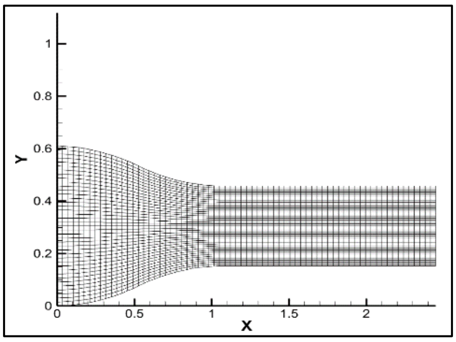

Figure 3. Curvilinear grids of Circular *arc*

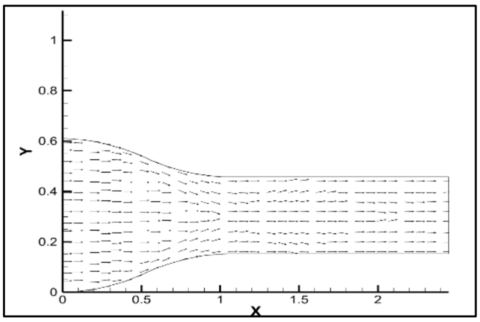

Figure 3. Flow vector pattern of circular-arc case

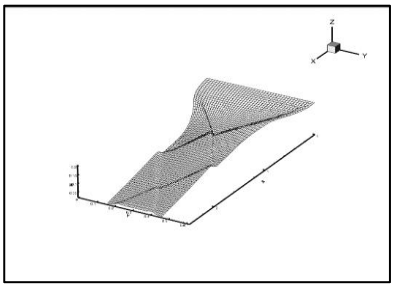

Figure 5. Perspective of water depth (real) of circular-arc case

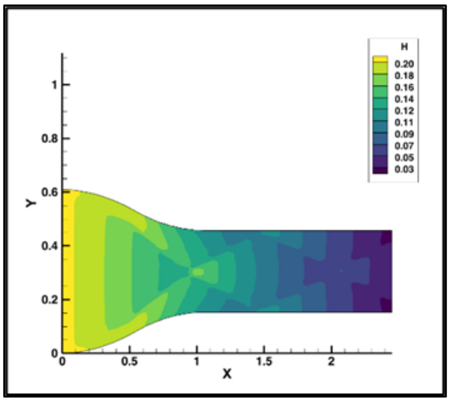

Figure 6. Contour of water depth for 10 seconds simulation

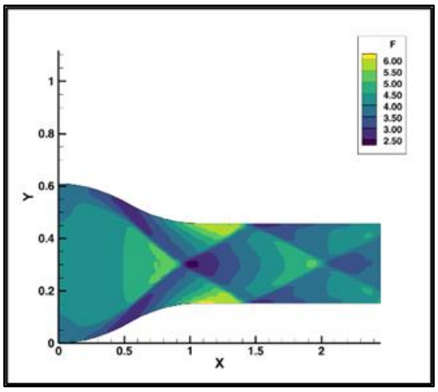

Figure 7. Contour of Froude number for 10 seconds simulation

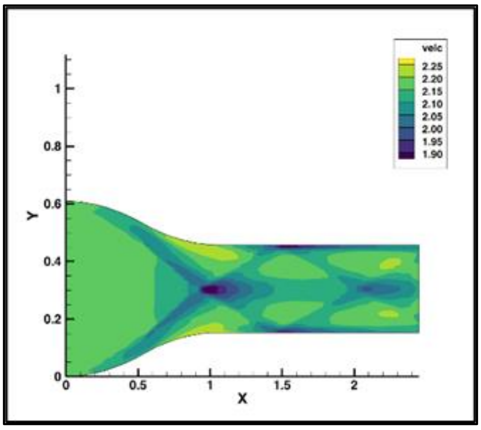

Figure 8. Contour of velocity flow for 10 seconds simulation

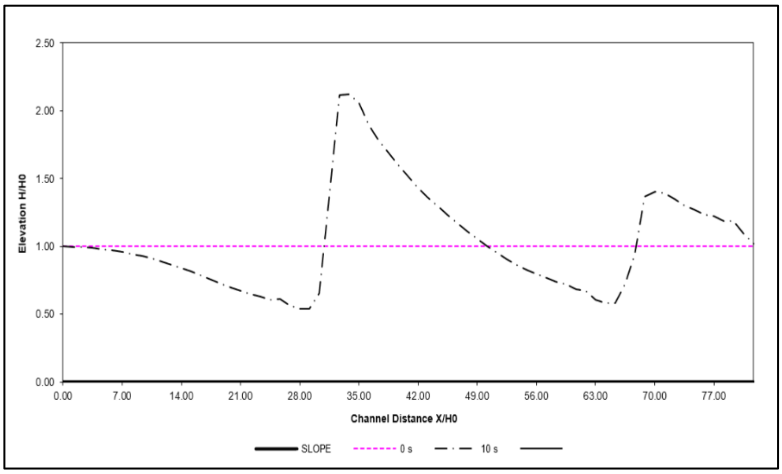

Figure 9. Distribution of water depth along center for 10 seconds simulation

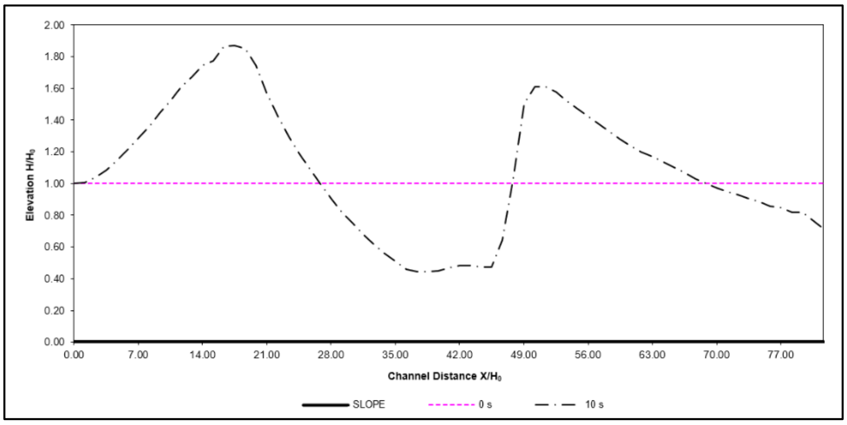

Figure 10. Distribution of water depth along the wall for 10 seconds simulation

3.2. Hydraulic Jump in 180º Curve Channel

Hydraulic jump case is divided into sub-critical part, super-critical part, and sub-critical part. The data used for this case are:

- $g = 9.81\ m^2/s$
- Length x Width = 300 m x 5 m
- $S_1 = 0.002$ (bottom channel slope from 0 to 100 m)
- $S_2 = 0.1$ (bottom channel slope from 100-200 m)
- $S_3 = 0.002$ (bottom channel slope from 200-300 m)
- $Q_0 = 10\ m^3/s$ (initial discharge)
- $H_0 = 2.34$ m
- n = 0.033 (manning)
- The computation time:
  - $\Delta t = 0.1$ second
  - Interval = 1000
  - Total time (time last) =1000

In this case, the downstream has water depth is equal $1.5H_0$. For the numerical solution, the channel is discretized with 61 grid point in x-direction and 11 grid point in y-direction.

The result of the computation as follows bellow.

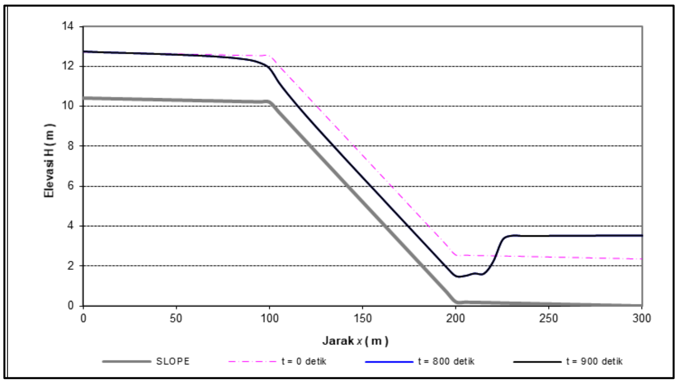

Figure 5. Longitudinal profile of 2-terrace hydraulic jump case

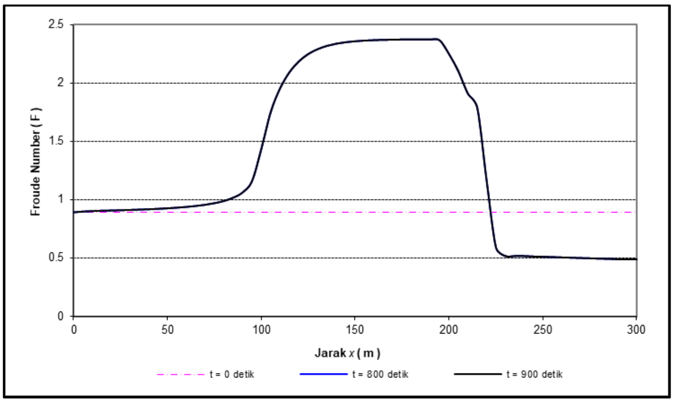

Figure 6. Longitudinal profile of 2-terrace hydraulic jump case of Froude number

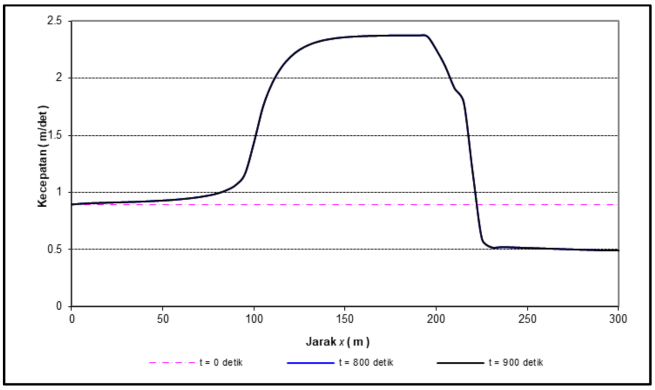

Figure 7. Longitudinal profile of 2-terrace hydraulic jump case of velocity flow

The criticality of the flow is influenced by the bed slope and flow velocity, where the steepness of the channel bottom causes the normal depth to be less than the critical depth so the value of the froude number is increasing.
The change in supercritical flow to subcritical causes a change in the water depth from low to high level. If the change occurs quickly on a relatively short distance will resulting a hydraulic jump.

3.3. Dam Break at Straight Channel

This Dam break experiments are carried out on 2D channels with a length=5 m and width = 1m and in the middle area there is a gate or gate that represents the dam. The initial conditions given are at the upstream part of 0.2 m and at the downstream part of 0.02 m. The running time of the program is 10 seconds at intervals of 1 second. The result of this dam break model is Mac Cormack FCT Scheme produce a good and smooth result.

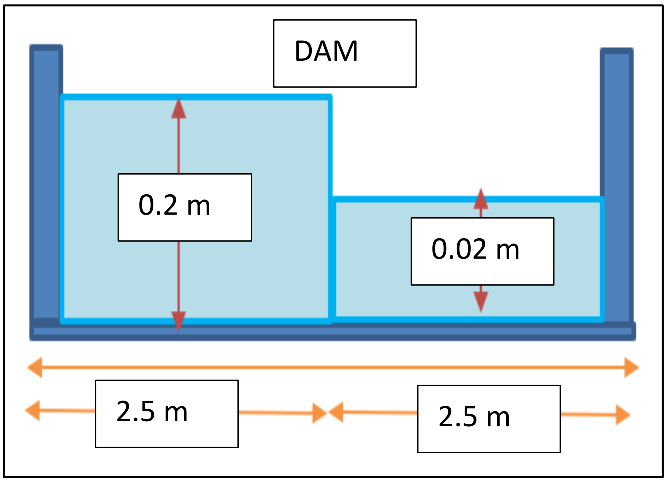

Figure 8. Dam break model

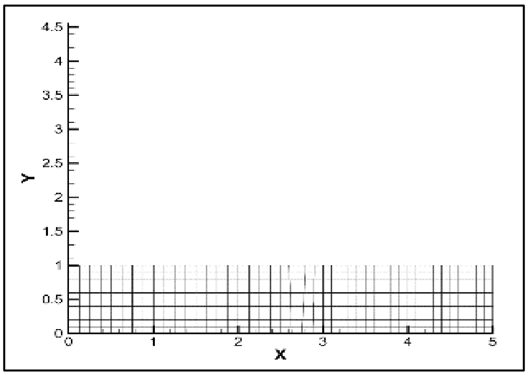

Figure 9. Dam break grid

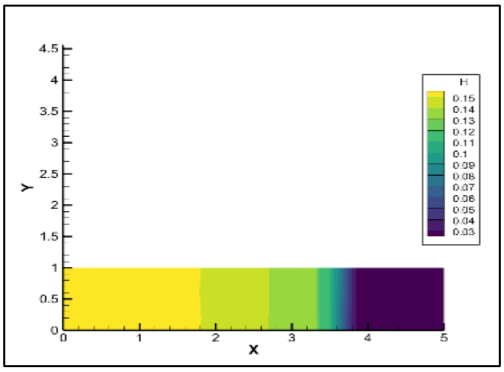

Figure 16. Water Depth

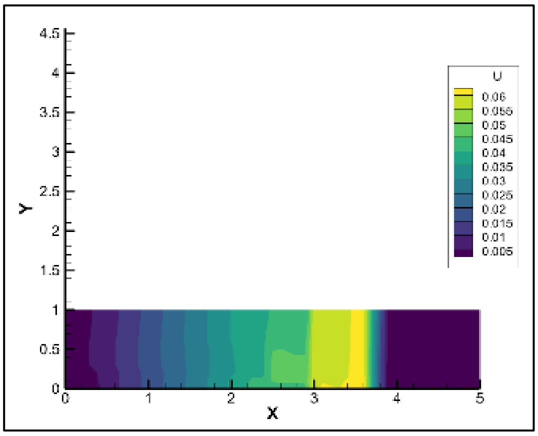

Figure 17. Water Velocity

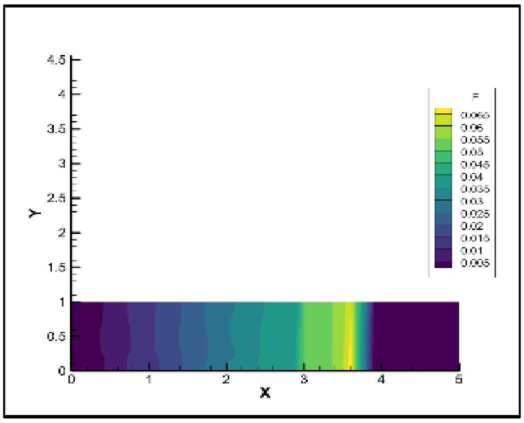

Figure 18. Froude number

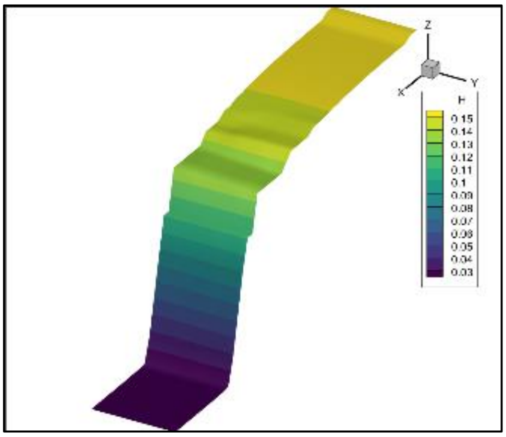

Figure 19. 3D View

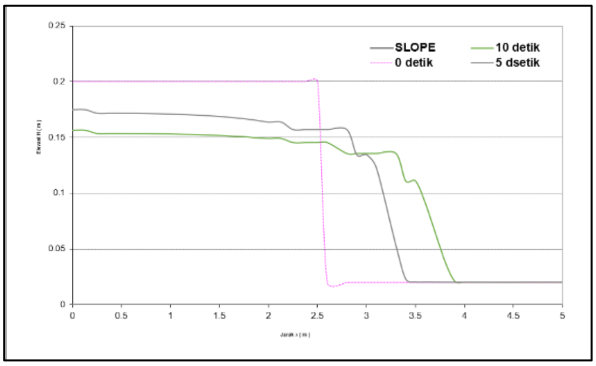

Figure 20. Long Profile of water elevation

4. Conclusion

Finite difference method used to solve the governing equation which is Saint Venant Equation and the scheme which used in this paper is Mac Cormack Flux Corrected Transport and to solve the discretization using finite difference method, it is necessary to transform the governing equation into the curvilinear coordinate. In this paper numerical method was using to simulate supercritical flows case. From the results, the simulation of the all cases using finite difference method and Mac Cormack FCT Scheme produce a good and smooth result. For further development of this study can be done using Mac Cormack FCT Scheme to simulate dam-break flows in more complex scenario or to simulate sub and supercritical flows in more channel modification.

5. Acknowledgement

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